Intergenerational income mobility: the role of the reference group. (Preliminary version - Please do not quote)

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Abstract

This paper formalizes the role of reference group income as a mechanism for inequality persistence between generations. Reference group theory suggests that culturally shaped processes may lead to a reduction (or an increase) in ambition among those that share them. As a result, lower-class backgrounds may be discouraged (or encouraged) by the relative deprivation effects, from making adequate mobility-enhancing investments. We develop a model in order to explain the role of reference groups in inequality persistence, and to explore how agents react to reference group income. The model confirms that reference groups could be a mechanism for transmitting inequality across generations, and demonstrates that the magnitudes and direction of these displacements depend on: (a) the composition of reference group: (b) intensity and functional form of income comparison, (c) ex - ante inequality between agents with different social origins and effort reward; and (d) the information about their peers and the past income mobility. This model is more general than previous models and its findings are in stark contrast to models based upon self-fulfilling beliefs and fatalistic prediction. Finally, this model represents a first bridge between reference group theory and aspiration failure models.

1 Introduction

This paper proposes a theoretical model to analyze the role of reference groups and the ex-ante inequality in intergenerational mobility. Our modeling exercise consists of finding the conditions under which individuals from lower class origin might be discouraged from making adequate mobility-enhancing investments, while individuals from higher classes origins might be more stimulated. The origin of that difference could rely in the the composition and trajectory of the reference groups, the relative effort rewards and ex-ante inequality among social origins.

Mainstream sociological theories have emphasized the role of social status, discriminatory beliefs and related cultural attitudes in the generation of persistent inequality between dynasties. The idea that reference groups play a crucial role in explaining income mobility has a long history in the social sciences but it has received less attention in the field of economics. Reference group theory suggests that culturally shaped processes may lead to a reduction (or an increase) in ambition among those that share them. People define different economic aspirations when their social origins and reference groups are heterogeneous. For example, poorer reference groups may transmit less ambition and taste for economic success to lower-class families than upper-class families with richer reference groups. As a result, the social origin and the reference groups could reduce the aspirations, and they could be an additional mechanism for the persistence of inequality between generations (Piketty,2000).

According to mainstream economics, persistence of inequality across generations can be explained by a combination of direct family transmission of productive abilities, endowments and parental investment in their children's human capital. The seminal theoretical contributions of Becker and Tomes (1979; 1986) suggest that intergenerational income correlation depends on various parameters, but the theory does not provide a consensual view about the magnitude of them (Solon, 1999).[1] Recent re-

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[1] Until 1992 the empirical evidence showed that the correlations between fathers' and sons' incomes were significantly positive, but quite short, indicating that family background was not a key factor to economic success (Becker and Tomes, 1986; Behrman and Taubman,1990). These findings contrast with Solon (1992; 2002), Zimmerman (1992), Björklund and Jäntti (1997), who suggest that previous studies have underestimated the intergenerational earning elasticity. Furthermore, the empirical literature on sibling correlation in earnings suggests a quite important role of family background and community origins. Solon (1999) argues that these findings suggest that most of the relevant factors about family transmission are uncorrelated with parental income. Chetty et al. (2014) use US administrative records to measure intergenerational mobility. Their estimates show that the intergenerational income elasticities in US is 0.45, but that magnitude is sensible to alternative specifications. Further,

search has found that intergenerational earnings elasticity is considerably higher than the most of the previous estimates had suggested. These findings raise new questions about what mechanisms could explain the low intergenerational mobility. Mitnik et al. (2013) uses the US General Social Surveys to analyzes intergenerational mobility. Their results suggest that the increase in income inequality in the US may account in part for the decline in intergenerational mobility. Chetty et al. (2014) use US administrative records to explore the factors correlated with upward mobility. They found that areas with less residential segregation, less income inequality, betters primary school, greater social capital and family stability have higher intergenerational mobility.

Bourguignon (Ferreira and Walton, 2007) suggest that socio cultural inequalities can partly explain inequality persistence and they emphasize that further economic research, both theoretical and empirical, in this area is needed. The seminal paper of Piketty (1998) is a notable exception within economic research.[2] It suggests that status motives amplify the inequality between agents with different social origins when the impact on the economic success of social origins is high compared to the effect of effort and ability. Piketty's model provides a general framework which could describe two socio cultural channels of inequality persistence: reference group theory (Boudon, 1974) and the statistical discrimination theory (Bourdieu and Passeron, 1964; 1974). However, the status motives used in Piketty's model provide a better basis for the discrimination theory. While, the reference group mechanism has not been considered in detail in this model. His utility function does not include the way in which people care about their relative position with respect to a referent point. Additionally, this framework does not allow us to explore how agents react to the composition of their reference group and the conditions under which reference groups might decrease (increase) income mobility. People care a lot about their relative position with respect to their reference group, even if this has no direct effect on their opportunities. Both the Prospect Theory and the standard Theory suggest alternative assumptions to consider relative concern in the utility function. However, this issue has received very little attention in Theoretical Economic literature, and much of our knowledge of the importance of interpersonal comparison comes from economics empirical literature

they found that on average a 10 percentile increase in parent income is associated with a 3.4 percentile increase in a child's income.

[2]Another important precedent is Borjas (1992), who proposes a model to analyze the link between socioeconomic performance and the external effect of ethnicity through ethnic neighborhoods. On other hand, Akerlof (1997) models the role of social distance on social decision, and analyzes the mobility between social positions.

or from Sociology. In particular, very little has been written on the effect of relative concern on intergenerational income mobility.

This paper attempts to build a bridge between intergenerational mobility and the relative concern literature. The central issue examined in this paper is the role of reference group income as a mechanism behind the intergenerational transmission of economic inequality. This paper provides a common framework to explain the role of reference groups on income mobility and to identify the incidence of ex-ante inequalities and reference group composition on effort decisions. Furthermore, it allows us to explore the conditions under which richer reference groups might increase income mobility or amplify inequality persistence.

Our approach incorporates the idea that the agent objective function considers the "self-perceived valuation of his relative position in his reference group". The composition of reference groups defines a reference income level, and agents care about the gap between their income and their reference income. We model rational agents with two different social origins, who choose the levels of effort which maximizes their expected utility. Furthermore, they know the relative importance of effort and predetermined factors for achieving economic success. Based on the alternative assumptions of standard Theory and Prospect Theory, we model the effect of reference group income on effort decision. In the first step, we assume that agents know the composition of their reference group and they have perfect information about their reference group income (forward-looking agents). In the second step, we assume that agents have imperfect information about the expected effort of their reference groups and that they base their choices on a *priori* beliefs about the probability of economic success in different social origins. Beliefs are updated according to Bayes' rule, implying that past mobility affects the expected income of the current generation. This framework allows us to derive long term effort equilibrium and to examine the effect of relative concern on the dynamic of intergenerational mobility.

Our model allows us to analyze the role of reference group on effort decision and income mobility in detail. We consider four aspects of reference groups separately: First, the intensity of relative concerns and the implications of standard assumption or Prospect Theory on effort decisions. Second, the incidence of the composition of reference groups for agents with different social origins. Third, the incidence of the beliefs about the peers' expected effort, and we consider how it could be affected by the past mobility experience in the society. Fourth we analyze how ex-ante inequal-

ity between agents with different social origins affect effort decision through relative deprivation.

The results that emerge from our model confirm that the reference group, through relative income effect and aspiration conformation, tends to amplify (or reduce) the inequality of economic success between agents with different social origins. The individual characteristics of relative concern, the composition of reference groups and the past mobility trajectories for agents with different social origins could easily generate multiple equilibriums in effort levels. Consequently, even when all agents could be identical in their abilities, their effort levels differ in the long term, which affect long term income mobility. Results suggest that the magnitude and direction of this effect depend on 4 key issues: (a) the composition of reference group being relevant regardless of inheritance patterns; (b) assumptions about the functional form of relative concern being keys issues to answer regarding the effect of reference groups on income mobility, where standard assumption or Prospect theory explain situation in which the income mobility would be very different; (c) ex - ante inequality and relative effort rewards; (d) expected effort beliefs and past mobility perceptions.

Five pieces of evidence lead us to think that this model is of some importance. First, empirical evidence suggests the relevance of relative concern on human motivations (Frank, 2005) and economic satisfaction (Card et al., 2012). Furthermore, experimental results support the reference-dependent utility suggested by Prospect Theory (Kanheman and Tversky, 1979). Second, the model could help to explain why societies with more equality in income distribution and less polarization show higher intergenerational mobility (Solon, 2002; Mitnik et al., 2013). Third, it contributes explaining the evidence about heterogeneous aspirations and adaptive preferences hypothesis (Festinger, 1975; Sen 1985a; 1985b; Elster, 1985; Clark, 2009). Fourth, this model could help explaining situations of low mobility for certain social groups and contribute to explain why agents with a similar familiar background and abilities obtain different economic achievement. Fifth, relative earning information is relevant in explaining effort worker decisions (Huet-Vaughn, 2013).

Finally, because the impact of reference group on income mobility is through the formation of aspirations, this model establishes a first bridge between reference group theory and the aspiration model of Genicot and Ray (2010). People form economic aspirations based on their past experience and their interactions with their reference group (Appadurai, 2004; Genicot and Ray, 2010). Therefore, the model is useful to

explore the conditions that leads to aspirations failures. First, a poorer reference group could reduce an agent's economic aspirations and could lead to low effort levels. In this case, agents with low social origin do not include agents with high social origin, which leads to aspiration failure type I (Ray, 2006). Second, under certain circumstances, previous inequality and relative concern could lead to low aspiration. In this case agents with low social origins include individuals from richer origin in their reference group but the relative costs of effort is too high, and the relative reward too low. As a result they reduce their aspirations and effort level in order to avoid frustration, which Ray (2006) named aspiration failure type II. However, under certain conditions reference groups could reduce inequality of economic success, which is in stark contrast to other models of inequality based upon self-fulfilling beliefs and fatalistic prediction.

The issue of intergenerational mobility is still one of the most controversial issues, both in political debates and in academic research by social scientists. Piketty (2000) argues that socio cultural inequalities could generate extra inequality persistence, where intergenerational mobility would be inefficiently low. In this context, appropriate corrective policies (or alternative wealth distribution) could raise intergenerational mobility and output at the same time. As a result, these models break with the equality efficiency trade – off, therefore corrective policies can raise intergenerational mobility and improve efficiency simultaneously. Piketty's conclusions are ambiguous when persistence is explained by reference group theory. In this case policy intervention could be driven solely by distributive justice considerations because individual do not respond to economic incentives. In contrast, Ray (2006) argues that it is perfectly possible for an unequal society to create local attainable incentives among the poorest individuals. Affirmative action and public education may be policy tools that could be used to create higher local connectedness and to affect aspiration conformity. Our model allows us to advance in this discussion. The ability to better understand this phenomenon will increasingly allow researchers to make public policy recommendations based on new theoretical models and new empirical applications.

The rest of this paper is organized as follows. The next section reviews the micro foundation of status concern (2.1) and the main theoretical advances in this topic from the Economic perspective (2.2). The third section focuses on the role of income comparison and its implications in terms of effort decision and income mobility when we assume forward-looking agents. The fourth section considers backward-looking

agents under imperfect information and introduces an updating beliefs rule to describe the long term effort equilibrium. This allow us to examine the effect of relative concern in the dynamic of intergenerational mobility. Finally we conclude.

2 RELATIVE CONCERN FROM ECONOMIC PERSPECTIVE

2.1 Microfoundations of relative concern

Postlewaite (1998) and Frank (2005) suggest that evolutionary theory provides a strong argument for an innate concern for relative standing. In this case, agent's relative concern is explained by competition for relative position in their evolutionary past. The evolutionary explanations argue that when agents have achieved relatively high position in the past, they will have better opportunities to reproduce. Hopkins (2008) points out that there are at least three different evolutionary explanations. The "rivalry story" (the success of others agents reduces own opportunity), "information story" (the experiences and success of other agents is useful information about potentially profitable activities) and "perception story" (because preferences are incomplete, relative comparison is a fundamental psychological mechanism to evaluate goods, resources, or opportunities).[3]

Finally, aspiration conformation may offer an alternative explanation of relative concern. The anthropologist Appadurai (2004) suggests that aspirations are always formed in interaction and in the thick of social life. It that, individual goals don't exist in social isolation, they depend on distribution of income and wealth.

2.2 Exploring the role of status in economic Literature

Sociologists have a long standing interest in the concept of social status to study the social interactions (Weber, 1922). However, this concept has received little attention

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[3]An alternative explanation would be that relative concern arises from current social arrangements and not have to arise from social preferences and past arrangements. In this case relative concerns do not arise because agents have competitive social preferences, but the nature of economic competition of the institutions lead that individuals make relative comparison. In this case, although agents only care about themselves, relative concern is instrumental to material benefits (Hopkins;2008).

in economic. Postlewaite (1998) and Weiss and Fershtman (1998) discuss how economic has introduced status. People's ranking could be an argument of the utility function, even if people derive no clear economic benefits from them.[4] Sen (1985; 2000) and Frank (1985; 2005) argue the relevance of status for well-being. People care about their relative performance compared to others. As a result, the individuals' self-assessment of her relative position should be considered as an argument of her utility function (Postlewaite,1998; Weiss and Fershtman, 1998).

Social status is the relative position of individuals in a given social group. People could have different status rankings. It will depend on whom they associate with the reference group in which each individual is evaluated. The reference group could be their friends, job's colleagues or society at large. As a result, when people compare with others they assign different weights to the individuals of their reference group (Weiss and Fershtman, 1998; Clark, 2008; Van Praag and Ferrer-i-Carbonell, 2008; Clark and Senik, 2010). However, the empirical literature regarding the process of selection of reference group is inconclusive. Clark and Senik (2010) suggest that this process seems to be partly endogenous and agent's benchmarks are related to the type of regular social interactions of them. However their findings are not consistent with this hypothesis.

Because the person's willingness to pay for social status could be very high, this issue is also relevant to understand important aspects of economic behavior. Although, it is difficult to accurately establish the importance of status on economic performance, it would be expected higher significance when the markets are thin. Hence, status may act as a social reward or punishment and it could be a corrective mechanism for some market failures such as externalities, transaction cost or monitoring problems. In this case, social status could raise efficiency. However, some approaches emphasize the role of status as an instrument to restrict entry and impose modes of belief. In this case, it becomes a mean to maintain advantages of privileged groups. As a result, status changes agent's behavior, may affect efficiency and allocation of outcome. However, the direction of these effects is not clear (Weiss and Fershtman, 1998, Frank, 2005; Heffetz and Frank, 2011).

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[4]One central issue is whether status is a direct argument of the utility function or its relevance is only instrumental. In this paper we assume that status has intrinsic value. In the second interpretation, status is relevant because it indirectly affects their opportunities and could be interpreted as an investment decision. In this case, status could be analyzed within the traditional economic paradigm, which assumes agents optimizing with stable preferences (Postlewaite,1998).

2.3 Economic modeling of relative income concern

Hopkins (2008) summarizes the main models of relative income concern and distinguished two groups of models. First, the author identifies a set of models which support relative concern based on three foundations: envy, pride or compassion. Second, others groups of studies support relative income concern based on inequality aversion.[5] Because last models do not consider relative care with respect a reference point, this section focuses on the first groups of models. According envy effect, the utility of an agents declines when an increase in the income of people richer than they occur (namely $\frac{\partial U(.)}{\partial dy} > 0 \, if \, y^{RG} > y$, where $U(.)$ is the utility function, y^{RG} and y are referent group income and agent's income respectively and $y^R = y - y^{RG}$). Duesenberry (1949) argues that poorer individuals are negatively influenced by the income of their richer peers, while the opposite is not true ($\frac{\partial U(.)}{\partial y^R} > 0 \, if \, y^{RG} > Y, \frac{\partial U(.)}{\partial yR} = 0 \, if \, y^{RG} < y$). However, literature suggests the pride effect (also named competitiveness), which assumes that the utility of an agent reduces with any improvement for others income ($\frac{\partial U(.)}{\partial y^R} > 0$). Secondly, some authors assume that an agent is better when there is an improvement in the income of those agents below than he ("compassion effect" $\frac{\partial U(.)}{\partial yR} < 0 \, if \, y^{RG} < y$).[6]

The expected utility approach for decision-making under uncertainty (Prospect theory) developed by Kahneman and Tversky (1979), provides some issues for relative concern modelization. It suggests that welfare depends more on deviations from a reference level than on absolute levels. Negative changes generates a higher impact on utility than gains of equal magnitude (loss aversion), and that preferences could be convex in the loss area (principle of diminishing sensitivity). Finally, this theory suggests that individuals make decisions based on subjective probability assessments. According to Tversky and Kahneman, (1991) reference group income provides a nat-

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[5]In this case, agents dislike difference between their income and that the others. The extensive literature on social preferences supports these assumptions. The original model of Fehr and Schmidt (1999) assumes that agents dislike of other having more (envy) but low income for others reduces their utility (compassion).

[6]There are models that combine these effects on the basis of different functional forms. Some models assume that utility includes a relative component where agent compares its income with the average income of other. These models is called mean-dependence model (Duesenberry,1949; Abel,1990; Boskin and Sheshinski,1978; Clark and Oswald,1996; 1998; Van Praag 2011). Other authors include relative income concern based on rank (Layard,1980; Robson,1992; Clark, Kristensen and Westergård-Nielsen,2009a; 2009b). Hopkins (2008) demonstrates that mean dependent models are a special case of Fehr and Schmidt (1999) model, where there is no compassion, and pride effect is as strong as envy effect.

ural reference point to income comparison. However, there is a current debate on what income level taken as a reference, as well as the possibility that individuals with similar characteristics may present differences in their reference income (Vendrik and Woltjer, 2007; Clark and Senik, 2010).

Based on prospect theory assumptions, Ray and Genicot (2010) propose a modelization of aspirations formation. They assume economic aspiration as a reference point, which depends on own historical living standard but also on the lifestyle of others. As a result, they suggest a relationship between the formation of aspiration and distribution of income. In this relationship is central the "aspiration window", which defines the individual's cognitive world (Ray, 2006; Mookherjee, Napel and Ray.. 2010).

In summary, most of the studies assume $\frac{\partial U(.)}{\partial y^R} > 0$ and there is a consensus on the asymmetry in the income comparison with respect to reference income. In general, models assume the standard assumption of diminishing marginal utility of relative income when $(\frac{\partial^2 U(.)}{\partial^2 y^R} < 0 \, if \, y^{RG} < y.$). However, there is less agreement on the sign of the second derivative with respect to relative income, for those individuals with relative deprivation. Vendrik and Woltjer (2007) argue that objective function could be convex or concave in relative income, for those agents with negative relative income. On one hand, the standard assumption of diminishing marginal utility of income in neoclassical theory suggests concavity of objective function in relative income $(\frac{\partial^2 U(.)}{\partial^2 y^R} < 0 \, if \, y^{RG} > y)$. On the other hand, if relative income concern is based on comparison to the social reference income, is plausibly argues in prospect theory to exhibit convexity of utility function, reflecting diminishing marginal sensitivity to larger deviations from the reference group income $\frac{\partial^2 U(.)}{\partial^2 y^R} > 0 \, if \, y^{RG} > y)$. These assumptions about relative concern (and their empirical support) allow us to discuss the role of reference group on income mobility.

3 A MODEL OF EFFORT CHOICE WITH REFERENCE GROUP

3.1 The agent's objective function

In order to discuss how optimal effort decisions are affected by income comparison, an additional argument in an individual's utility function is included in the standard basic

model: the "self-perceived valuation of its relative position". Therefore, the objective function of agent i is given by:

$$U_i(y_i, y_i^R, e_i) = (1-\alpha)y_i - \alpha G(y_i^R) - C(e_i) \tag{3.1}$$

where U_i is the utility function for agent i. Agents enjoy their income (y_i) for consumption reasons, dislike effort e_i because they enjoy leisure (agents perceive that effort is a cost defined by the function $C(e_i) = e_i^2/2a$, with $a > 0$).[7] Agents care about their relative deprivation (RD) which arises from a comparison between their income and that their reference group, and they dislike unfavorable income comparisons. The function $G(y_i^R)$ is an attempt to formalize the discussion of how reference group income and RD affects an agent's utility, where y_i^R represents the difference between his own income (y_i) and expected reference group income (y_i^{RG}), ($y_i^R = y_i - y_i^{RG}$). [8]

For simplicity reasons, first we assume that utility function is additively separable, and that status motive is a direct argument of the utility function due to its intrinsic value, where, $0 < \alpha < 1$ measures the extent to which agents care about it. Following the assumption discussed in section 2.3, $G(y_i^R)$ is defined as:

$$G(y_i^R) = \begin{cases} G(y_i^R) = G(y_i^R) > 0; G_{y_i^R}(.) < 0; G_{y_i^R y_i^R}(.) > 0 & if\, y_i^R < 0 \\ G(y_i^R) = c & if\, y_i^R \geq 0 \end{cases} \tag{3.2}$$

As in previous studies, we assume an asymmetry in the income comparison.[9] The function $G(y_i^R)$ incorporates envy effect and concavity of relative income when $y_i^R < 0$

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[7]Because this paper focuses on the incidence of relative income on effort decision, with the aim of simplifying, it assumes a lineal relationship between the absolute income and utility. However, other approaches assume a non lineal relationship, and they explain their implications in terms of income mobility (Lewis and Ulph, 1998; Antman and McKenzie; 2007; Carter and Barrett; 2006).

[8]We assume a cardinal perspective of relative income concern. This decision is based on previous papers. Furthermore, allow us to o build a bridge between relative concern literature and aspiration models. As is noted in Bilancini and Boncinelli (2008), cardinal and ordinal approaches have different implications. Nevertheless, it must be emphasized that assumptions about second and third derivatives of G(.) incorporate ordinal concern (Kolm, 1976a; 1976b).

[9]Other studies have already used this assumption. Stark et al. (2011) used the same assumption to formalize the link between human capital choices and social location choices. Bowles and Park (2005) used it to modeling the "Veblen effect". Ray and Genicot (2010) also suggest upward looking aspirations formation to describe the relationship between social interaction and aspiration formation. Dalton, Ghosal and Mani (2014) use a similar framework to explain aspiration failure. Dusenberry (1949) postulated and tested the hypothesis that relative income comparisons are asymmetric. Finally, this assumption is supported by Bowles and Park (2005), Stuzter (2004) and Ferrer-i-Carbonell (2005).

(Hopkins, 2008). Agents care about having a low gap between their income and their reference group income. Furthermore, this function is more general than previous studies because $G(y_i^R) = c$ when $y_i^R \geq 0$, which leaves open the possibility that agent - relative concern is supported by "pride effect" or "compassion effect". Note that the asymmetry in the income comparison is also considered in the differences in the derivatives, where $G_{y_i^R}(y_i^R) < 0$ and $G_{y_i^R y_i^R}(y_i^R) > 0$ when $y_i^R < 0$ and $G_{y_i^R}(y_i^R) = 0$ when $y_i^R \geq 0$. Namely the disutility is constant with respect to the relative income when the pride or compassion effect exist, but the marginal utility increase when the envy effect operates. Following both theoretical and empirical literature, this assumption recognized that agents are upward looking when making comparisons and that the envy effect dominates relative comparison. With the aim of simplifying, first we assume that pride effect on relative concern dominates, as a result $c \leq 0$.

3.2 Social origin and expected income

We assume an economy, in which agent's income is a random variable and there are two possible income levels: y_0 and y_1 ($0 < y_0 < y_1$ and $\Delta y = y_1 - y_0$). That economy is made up of a continuum of agents $I = [0;1]$, who can be divided in two social backgrounds: lower class origin (I_L; i.e. whose parents' income level was y_0) and upper class origin (I_U; i.e. whose parents' income level was y_1). The probability that agent i obtains a high income level depends positively on their ability (β), their effort (e_i) and luck π. Furthermore this probability is conditioned by social origin and it is given by:

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\begin{aligned}
Pr(y_i = y_1 | I_L) &= \pi + \theta \beta e_i \\
Pr(y_i = y_1 | I_U) &= \pi + \Delta \pi + \theta \beta e_i
\end{aligned}
\tag{3.3}
$$

where, $Pr(.)$ defines the probability of the event in brackets occurring and $\Delta \pi$ measures previous inequality between agents with different social backgrounds.[10] Meanwhile, $\theta > 0$ is the same for all agents and measures the extent to which higher effort and higher ability can translate into higher probabilities of high income. Because they receive inheritance from previous generations, for the same effort the expected probability of economic success is higher for agents with origin I_U than for those with

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[10]This parameter could explain the inequality of family transmitted human capital and/or inequality of collateral in case of credit constrains (Piketty,1998).

origins I_L. Agents have perfect information about the parameters that determine the probability of economic success (π, $\Delta\pi$ and θ) (*Assumption A.I*). As a result, the expected income for those with lower class origins and higher class origin is respectively defined as follow:

$$E(y_i|I_L) = (\pi + \theta\beta e_L^b)y_1 + (1 - \pi - \theta\beta e_L^b)y_0$$
$$E(y_i|I_U) = (\pi + \Delta\pi + \theta\beta e_U^b)y_1 + (1 - \pi - \Delta\pi - \theta\beta e_U^b)y_0 \tag{3.4}$$

We assume that individual effort levels are not publicly observable, everybody expects that agents with lower class origins put effort e_L^b and those with upper class origins put effort e_U^b (*A.II*). Ex-ante agents do not have any information about their ability β_i and they assume the mean β_M of the ability distribution $f(\beta_i)$, with $0 < \beta_i \leq B$ (*A.III*). Following Piketty (1998) we make two natural assumptions. There is an exogenous maximum effort level $\bar{E} > (1 - \alpha)a\theta\beta_M\Delta y$ (*A.IV*). Further, we assume $\pi + \Delta\pi + \theta B\bar{E} < 1$ (*A.V*). Finally, we assume that the expected income for agents with higher class origin is at least equal to the expected income for agents with lower class origin, because $\Delta\pi$, $(\pi + \theta\beta E)y_1 + (1 - \pi - \theta\beta E)y_0 = Max(E(y_i \mid I_L) = E(y_i \mid I_U)$ (*A.VI*). This assumption implies that the effect of the differential in expected effort on economic success never outweighs the effect of previous inequality.

3.3 The reference group income

Now we introduce an analytical form to introduce reference group. The idea is that the composition of reference groups defines a reference income level and agents care about the gap between their income and their reference income. The set of agents $P_i(I_U) + (1 - P_i)I_L$ integrates the reference group of agent i. Each agent i knows his P_i, which is a random variable with the distribution function $F(P_i)$ for all $P_i : 0 \leq P_i \leq 1$.[11] Agent i with social origin I_L compares only with his peers when $P_i = 0$, and he compares only with upper-class agents when $P_i = 1$. As a result, the expected income of the reference group, y^{RG}, is defined as $y^{RG} = P_i(E(y|I_U)) + (1 - P_i)E(y|I_L)$.

The expected relative deprivation depends on the expected income for agents with different backgrounds and on the composition of reference groups. Consider first the

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[11]This assumption is simplistic but it is in agreement with the current empirical findings about the individual's group reference choice. This assumption would be lifted in future research to analyze the role of hereditable reference group or an endogenous choose of reference group. Falk and Knell (2004) propose a model where the agents optimize their choice between alternatives reference standards.

case of agent i with lower-class origin (I_L). The ex-ante expected relative deprivation is defined as:

$$E(y_i^R \mid I_L) = E(y_i \mid I_L) - E(y_i^{RG}) = \Phi(e_i, e_L^b, e_U^b, P_i)$$

$$\underbrace{P_i}_{Composition} \quad \underbrace{(E(y \mid I_L) - E(y \mid I_U))}_{\substack{Expected\, income\, gap \\ between\, agents\, I_L and\, I_U}} \quad + \quad \underbrace{E(y_i \mid I_L) - E(y \mid I_L)}_{\substack{Expected\, income\, gap \\ of\, agent\, i, with\, his\, peers}} \qquad (3.5)$$

where $E(y_i \mid I_L)$ is the expected income of agent i, given that he is I_L, and $E(y \mid I_L)$ is the expected income for agent with origin I_L, which was defined in equation 3.3. Relative deprivation is composed by three terms, the wide of reference group (P_i), the expected gap between agents with low and high social origin $(E(y_i \mid I_L) - E(y_i \mid I_U))$, and the expected gap with respect peer income. Observe that relative deprivation has a random component P and a hereditable component, the expected income conditional to the origin.

On the other hand, we assume that $P = 1$ *(A.VII)* for agents with upper-class origins, which is consistent with the previous literature that income comparisons are not downward-looking.[12] We assume that social comparisons are upwards, which in this model represents that the richest agents only compare with his peer. For agents with origin I_U, the expected relative deprivation is defined as:

$$E(y_i^R \mid I_U) = [E(y_i \mid I_U) - E(y \mid I_U)] \quad (3.4.b)$$

Observe that if $e_U^b < e_L^b$, we arrive at the conclusion that $(E(y_i^R \mid I_L) \leq E(y_i^R \mid I_U))$, namely, regardless of the value of P_i relative deprivation is equal or higher for agent with origin I_L than for agents with origin I_U.

3.4 An agent's effort decision process

We assume that agents live for one period, are rational and act to maximize their expected utility based on the parameters of the Economy and their beliefs. As a benchmark, consider an agent optimization, where e_L^b and e_U^b are exogenous and agents know

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[12]This assumption is not essential. The conclusions of section 3.4, which assume forward-looking agents, do not change if we assume that $0 \leq P \leq 1$ for agents with upper-class origins.

their values (each agent takes others' effort as given). Then they know expected income of their reference group. For an agent i with lower-class origin, the optimization problem is defined as:

$$\begin{cases} Max E\left[U_i(y_i, y_i^R, e_i) \mid I_L)\right] = (1-\alpha)E\left[y_i \mid I_L\right] - \alpha E\left[(G(y_i^R \mid I_L))\right] - C(e_i) \\ \qquad S.a. E(y_i^R \mid I_L) = \Phi(e_i, e_L^b, e_U^b, P_i) \end{cases}$$

(3.6)

The first order condition is.

$$e_{Leq}(P_i) = \begin{cases} e_{Leq}^* = (1-\alpha)a\theta\beta_M\Delta y & if\, E(y^R \mid I_L) \geq 0 \\ e_{Leq}^{**} = e_{Leq}^* - \alpha a\theta\beta_M\Delta y G_{y^R}(y_{Leq}^R \mid I_L) & if\, E(y^R \mid I_L) < 0\ \&\ e_{Leq}^{**} < \bar{E} \\ e_{Leq} = \bar{E} & if\, e_{Leq}^{**} \geq \bar{E} \end{cases}$$

(3.7)

All agents with the same reference group will choose the same optimal effort. Namely, agents with origin I_L and the same P_i, will choose the same optimal effort, where the index i identifies the reference group composition $e_{Leq}(P_i)$. However, agents with the same origin may choose different long term effort level because they vary with the composition of the reference group. This result deviates from Piketty (1998), where all agents with the same origin arrive to the same long term effort level.

For agents with origin I_U the optimization problem is defined as:

$$\begin{cases} Max E\left[U_i(y_i, y_i^R, e_i) \mid I_U)\right] = (1-\alpha)E\left[y_i \mid I_U\right] - \alpha E\left[(G(y_i^R \mid I_U))\right] - C(e_i) \\ \qquad S.a. E(y_i^R \mid I_U) = \Phi(e_i, e_L^b, e_U^b, P_i) \end{cases}$$

(3.6.b)

As a result, the first order condition is. (7.b).

$$e_{Ueq} = \begin{cases} e_{Ueq}^* = (1-\alpha)a\theta\beta_M\Delta y & if\, e_U^b \leq e_{Ueq}^* \\ e_{Ueq}^{**} = e_{Ueq}^* - \alpha a\theta\beta_M\Delta y G_{y^R}(y_{Ueq}^R \mid I_U) & if\, e_U^b > e_{Ueq}^* \\ e_{Ueq} = \bar{E} & if\, e_{Ueq}^{**} = \bar{E} \end{cases}$$

(3.7.b)

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The second order condition $\left[-\alpha(\theta\beta_M\Delta y)^2 G_{y^R y^R}(y_{eq}^R \mid I_L) - \frac{1}{a} < 0\right]$ holds because of the convexity of $G(y^R)$ (in accordance with Standard assumptions) and $c(e)$. Hence $e_{Leq}(P_i)$ and e_{Ueq} constitute optimum solutions.

Now we can discuss if relative deprivation generates differences in effort decision between agents with different social origins. First, note that when $\alpha = 0$ (i.e. without any relative deprivation), 3.7 and 3.7.b trivially define a unique equilibrium where all agents make the same effort. When $\alpha \neq 0$, the effort equilibrium depends on e_U^b and e_L^b.

First, there are two extreme cases: (a) if $Max(e_U^b, e_L^b) \leq (1-\alpha)a\theta\beta_M\Delta y)$, then $e_{Ueq} = e_{Leq}$. (b) If $e_U^b \geq \bar{e}_U^b$ and $e_L^b \geq \bar{e}_L^b$, then $e_{Leq} = e_{Ueq} = \bar{E}$. Both results predict the same effort for agents with I_L social origin and agents with I_U social origin. Furthermore, these results are consistent with "self fulfilling belief". In the first case, both for agents with origin I_L and agents with origin I_U, expected efforts are low, and they choose a low effort, in the second case, the expected efforts are high and they choose a high effort. Observe that, although $e_{Leq} = e_{Ueq}$, both scenarios establish that $E(y_i \mid I_L) < E(y_i \mid I_U)$ and $E(y_i^R \mid I_L) \leq E(y_i^R \mid I_U)$.

However, apart from these extreme cases $(Max(e_U^b, e_L^b) > (1-\alpha)a\theta\beta_M\Delta y)$ and $e_j^b < \bar{e}_j^b)$, any inequalities in expected efforts and relative deprivation yield different optimal choices. On one hand, when $e_U^b < e_L^b$ and $P_i \neq 0$, effort of agents with origin I_L is higher than effort of agent with origin I_U $(e_{Leq}(P_i) > e_{Ueq})$. Then the incorporation of relative deprivation increases the optimal level of effort chosen by an agent. This effect generates an upward jump in levels of optimal effort, when $E(y^R \mid I_L) \geq 0$ changes to $E(y^R \mid I_L) < 0$ (Observe that $\alpha \left[G_{y^R}(y^R) < 0\right]$, $G_{y^R y^R}(y^R) > 0$ and then $e_{Leq}^* = e_{Ueq}^* < e_{Ueq}^{**}$). Given that inequalities in expected effort $(e_U^b < e_L^b$ the effort of agents with I_L social origin equals the effort of agents with I_U social origin, only when $P_i = 0$. This condition represents a situation where agents with origin I_L only compare with their peers, and their expected effort (and income) is low.

Finally, on the other hand, if $e_U^b > e_L^b$, the differences in effort decisions depend essentially on P_i. There is a P^* such as $\frac{-P^*\Delta\pi}{1-P^*} = \theta\beta_M\left[e_L^b - e_U^b\right]$, which leads to $E(y^R \mid I_L) = E(y^R \mid I_U)$ and $e_{Leq}(P^*) = e_{Ueq}$. However, a more demanding reference group (higher P_i) increases effort of agents with origin I_L, and, $e_{Leq}(P_i) \gtreqqless e_{Ueq}$ when $P_i > P^*$. On the other hand, if P_i is lower than P^*, and agents with origin I_L compare mainly with their

peers, $e_{Leq}(P_i) < e_{Ueq}$ [13].

Forward-looking agents

The previous discussion is the most simple case: First, agents don't internalize ex-ante beliefs when they take effort decision; Second, it does not consider the interaction between individual effort decisions and the expected effort of their peers. An equilibrium is a vector of consistent effort decisions and efforts beliefs. To analyzes this case, we assume completed forward-looking agents, then they anticipate the actions of the others when they take effort-decisions (an extreme Cournot - Nash assumption satisfied). As a result, agents share the public beliefs about their expected income. Furthermore, we assume that agents decisions are composed of a two-step process. First, they identify their reference group income and expected relative deprivation (this step allows them to find the domain where the relative deprivation function $(G(.))$ works). In a second step, they maximize expected income, taking as given the reference group income and choosing their optimal level of effort.

We observe that, if we assume that ex-ante agents share the public beliefs about their expected income $E(y_i \mid I_J) = E(y \mid I_J)$, then $E(y_i^R \mid I_U) = 0$ and $E(y_i^R \mid I_U) \leq 0$.

First, observe that agents with origin I_U do not expect to face relative deprivation $(E(y_i^R \mid I_U) = 0)$. Therefore, for agents with origin I_U, the equilibria is $e_{Ueq} = (1 - \alpha)a\theta\beta_M\Delta y = e_{Ueq}^b$. Meanwhile, for agents with origin I_L, $E(y_i^R \mid I_U) \leq 0$ and the equilibria is defined by $e_{Leq}(P_i)$ and $e_{Leq}^b = \int p e_{Leq}(P)dp$. Regardless e_{Leq}^b, observe that $e_{Leq}(P_i) > e_{Ueq}$ when $P_i \neq 0$ and that $e_{Leq}(0) = e_{Ueq}$. The predictions are consistent with "the self fulfilling belief model" in a particular case, if each agent from lower-class backgrounds compares himself only with agents with the same origin ($P_i = 0$). Namely when agents share public beliefs and assume ex-ante that they belong to a reference group whose members are all I_L, they adopt a behavior that validates their reference group expectations. When the structure of reference groups is heterogeneous, agents with lower-class origins always have incentives to assume strategies to improve their opportunities to achieve a better life. [14]

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[13]Observe that when $\theta\beta_M\left[e_L^b - e_U^b\right] < \frac{-P_i\Delta\pi}{1-P_i}$, then $E(y^R \mid I_L) < E(y^R \mid I_U))$

[14]These results depends on the two step decision process, but general predictions do not change if we assume a one-step process. When agents do not share beliefs, for agents with origin I_L the equilibria is defined by $e_{Leq}(P_i) > (1 - \alpha)a\theta\beta_M\Delta y$ if $F(P_i) \neq 0$. Namely, reference group income always motives higher optimal effort of agents with low social origin when there is heterogeneity in the composition

An additional analytical result stems from differentiating implicitly in Eq. (3.7) to give the individual's effort response to an exogenous change in y^{RG} among agents with relative deprivation $(E(y_i^R \mid I_L) < 0)$.

$$de_{Leq}^{**}/dy^{RG} = \frac{\alpha a \theta \beta_M \Delta y G_{y^R y^R}(.)}{1 + \alpha a (\theta \beta_M \Delta y)^2 G_{y^R y^R}(.)} \, if \, E(y_i^R \mid I_L) < 0 \, and \, e_{Leq}^{**} < E \qquad (3.8)$$

This expression is always positive, because the denominator in Eq.3.8 is positive (by the second order condition) and the numerator is positive because convexity of G(.). The derivative is zero when effort reaches its maximum level $(e_{Leq}^{**} = E)$. As a result, for lower-class agents, a richer (or more demanding) reference group provides higher effort incentives. This effect is larger when agents care a lot about their relative position (high α) and when their marginal utility is more sensitive to changes in relative deprivation (high $G_{y^R y^R}(.)$).

Given agent i with lower-class origins, when $P_i \neq 0$, he has high economic incentives to increase the amount of his effort and the effect is stronger when P_i, $\Delta\pi, \Delta y$, e_U^b and e_L^b are higher. These incentives disappear if $E(y_i \mid I_L) \leq E(y_i^{RG})$, in this case $e_{Leq}^* = (1 - \alpha) a \theta \beta_M \Delta y$.

However, it may seem less intuitive that higher ex-ante inequality ($\Delta\pi$) always motives higher optimal effort. Previous literature found that the source of inequality explains, in part, preferences for income redistribution (Durante, Putterman, and van der Weele, 2014; Alesina and Angeletos, 2005). Experimental evidence shown that agents are willing to punish a unfair situations, even at some immediate cost to themselves (Henrich, Heine, and Norenzayan, 2010; Fehr and Hoff, 2011). Based on the same argument, it is possible to argue that people could changes their perception of the cost of effort because they think that the initial distribution is unfair. In our model, higher $\Delta\pi$ represents the stronger role of inheritance (unfair circumstance?) in the income performance, which could decrease motivation, inducing lower effort. In short, the assumptions presented above model the encouragement effect, but do not capture the frustration or complacency effect.

One point worth noting here is that these results depend critically on the assumption about the diminishing marginal sensibility of relative deprivation $(G_{y^R y^R}(.) > 0)$. However, evidence from prospect theory suggest that $G_{y^R y^R}(.) < 0$ when $E(y^R) < 0$,

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of their reference groups. But composition is still relevant because $de_{Leq}/dP > 0$. Furthermore, if $P_i = 0 \forall i \in I_L$, $e_{Leq} = (1 - \alpha) a \theta \beta_M \Delta y$.

which reflects diminishing marginal sensitivity to larger deviations from the reference group income (see section 2.3). This assumption is also supported by Kuziemko et al. (2013), who argues that in presence of last-place aversion, the utility of the agents in the bottom of the income distribution may be convex with respect to relative position. In this case, when $G_{y^R y^R}(.) < 0$ and $\alpha \theta \beta_M \Delta y \left[G_{y^R y^R}(.) < \frac{1}{a} \right]$, in a range of values of y^R, the optimality condition still hold and then de_{Leq}^{**}/dy^{RG} will be negative.[15] In this case, more demanding reference groups lead to lower effort. Both results are predictable. Marginal utility measures how much extra utility agents gain from their higher relative income ($U_{y^R} = \alpha G_{y^R}$). The function measures how much marginal utility will change in response to a change in the level of relative deprivation (marginal sensitivity). When $G_{y^R y^R}(.) > 0$, higher relative deprivation increases the marginal utility of relative income, therefore motivation is higher. While $G_{y^R y^R}(.) < 0$, better relative income increases individual marginal utility, but this increase will be higher when y^R is lower. We have arrived at the following proposition:

> Proposition 1 When $E(y^R) < 0$, under additive comparisons and asymmetry in the income comparison, we arrive:
>
> (i) The relative deprivation effect increases the optimal level of effort chosen by an agent with relative deprivation compared to an agent without relative deprivation ($e_{Leq}^* < e_{Leq}^{**}$)
>
> ii. When the utility function is concave in relative income ($G_{y^R y^R}(.) > 0$), higher reference income always leads to additional effort ($de_{Leq}^{**}/dy^{RG} > 0$ with $e_{Leq}^{**} < \bar{E}$).
>
> iii. When the utility function is convex in relative income ($G_{y^R y^R}(.) < 0$), higher reference income always leads to lower effort equilibrium level $de_{Leq}^{**}/dy^{RG} < 0$ with $\bar{E} > e_{Leq}^{**} > e_{Leq}^*$).

Proof. direct from Eq. 3.8 and the functional form of $G(.)$

Assumptions about the sign of $G_{y^R y^R}(.)$ reflect the difference between the prospect and the standard theory, and are central in explaining the effect of reference groups, while allowing us to model both the encouragement effect and the frustration or com-

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[15]The implication of the expression $\alpha \theta \beta_M \Delta y \left[G_{y^R y^R}(.) < \frac{1}{a} \right]$ is that effort always is perceived as a cost. In other words, an increase of the marginal utility due to a decrease in the relative deprivation is lower than the increase of the marginal cost due to a higher effort.

placency effect. However, this specification is simplistic, because agents have fixed reaction rules when responding to changes in the reference group income.

3.5 An extension of the model

In the previous section, the relative component is only considered through relative (income) deprivation. However, reference group theory considers relative deprivation as a social and psychological experience, in which individuals take the standards of others individuals as a comparative "frame of reference". This defines "the patterns of expectations", but also the perception of "comparable sacrifice", in this way, it contributes to explain why attitudes differ among individuals (Merton, 1968). To address this issue we leave aside the additive comparisons assumption and include a more general function $G(y_i^R, e_i)$, which includes both relative income and relative effort (with respect to relative deprivation). This function incorporates the part of the cost of effort that is cultural and endogenous, whereas $C(e_i)$ is the part of effort that is exogenous to the relative situation. As a result, this function considers the way in which relative deprivation affects the perception of effort and how effort affects the sensitivity of relative deprivation.[16] In this way, we capture the idea that reference groups establish the "effort norm", which could affect individual motivations [17].

We include the function $G(y_i^R, e_i)$ in the agent's objective function and arrive:

$$U_i(y_i, y_i^R, e_i) = (1 - \alpha)y_i - G(y_i^R, e_i) - C(e_i) \tag{3.9}$$

---

[16]To make this assumption a little more concrete, consider an example of the function $G(y_i^R, e_i)$, $G(y_i^R, e_i) = g(y_i^R)v(e_i)$, with $g(y_i^R) > 0$, $g'(y_i^R) < 0$, $g''(y_i^R) > 0$ and $v(e_i) > 0$. Note that $v(e)$ is constant and equal 1 in the basic model. By making explicit assumptions about $v(e_i)$, we clarify the exact nature of the tastes required to explain a particular behavior. On the one hand, when the effort increases, the marginal utility of relative deprivation in the reference group will decrease. Namely $v'(e_i) < 0$, which implies $G_e(.) > 0$. On the other hand, the sensibility for relative deprivation might decrease with higher effort, if $v'(e_i) > 0$, which implies $G_e(.) < 0$. This function also captures how relative deprivation affects the perception of the cost of effort. For example, perception of the cost of effort could be lower when relative deprivation is low, because agents belief that reference group income is achievable outcome and they are motivated ($v'(e_i) > 0$). Alternatively, given a high relative deprivation, when effort is very high, agents could perceive that the goal is unattainable, they are discouraged and perceive that effort is less effective (or more costly, $v'(e_i) < 0$). Based on the notions of cognitive dissonance, relative deprivation and social comparison, Festinger (1957) argues that individuals compare their own input-to-output ratio with respect to a reference level. According to equity theory, if the comparison is perceived as "unfair", the individual may be motivated to change his behavior and restore his cognitive perception of equality (Adams, 1965).

[17]Kandel and Lazear (1992) or Akerlof and Kranton (2005), incorporate the notion of social norms and analyze how it affects work incentives.

Following the previous sections $G(y_i^R, e_i)$, is decreasing and convex in its first argument. However, in the second argument the situation is more flexible, and its functional form allow us to model different individual responses and include some convex part of the function G.

$$G(y_i^R) = \begin{cases} G(y_i^R, e_i) = G(.) > 0; \ G_{y_i^R}(.) < 0; \ G_{y_i^R y_i^R}(.) > 0 & if \ y_i^R < 0 \\ G(y_i^R) = c & if \ y_i^R \geq 0 \end{cases} \quad (3.10)$$

When we assume forward-looking agents and consider Eq. 3.9 in the optimization problem defined in Eq. 3.6 ,we can derive a new optimal effort conditions of agents with origin I_L and I_U.

$$e_{Leq}(P_i) = \begin{cases} e_{Leq}^* = (1-\alpha)a\theta\beta_M\Delta y & if \ E(y^R \mid I_L) \geq 0 \\ e_{Leq}^{**} = e_{Leq}^* - \alpha a\theta\beta_M\Delta y G_{y^R}(.) - a\alpha\left[G_e(.)\right] & if \ E(y^R \mid I_L) < 0 \ \& \ e_{Leq}^{**} < \bar{E} \\ e_{Leq} = E & if \ e_{Leq}^{**} \geq \bar{E} \end{cases}$$

(3.11)

$$e_{Ueq}^* = (1-\alpha)a\theta\beta_M\Delta y \ (3.11.b)$$

We assume that the problem has an optimal solution and the following second order conditions always hold:

$$-\alpha G_{ee}(.) - 2\alpha\theta\beta_M\Delta y G_{y^R e}(y_i^R, e_i) < \tfrac{1}{a} + \alpha\theta^2\beta_M^2\Delta y^2 G_{y^R y^R}(y_i^R, e_i) \quad (3.12)$$

As a result, $e_{Leq}(P_i)$ and e_{Ueq} constitute optimum solutions. The FOC remains unchanged for agents with lower reference group income, when $E(y^R \mid I_L) \geq 0$ "relative deprivation" has no effect on optimal effort level. However, this condition changes when $E(y^R \mid I_L) < 0$. If we only focus on interior solution, an agent with origin I_L will choose the level of effort e_{Leq}^{**}. In this case, the sign of $G_{e_i}(.)$ characterizes the agent's responses to reference group income and relative deprivation. The next proposition summarizes three types of individual's situations.

Proposition 2: When $E(y^R) < 0$, under non additive comparisons and asymmetry in the income comparison, we arrive:

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Positional self - encouraged agent: When $G_e(.) < 0$ (Condition I), relative deprivation increases the optimal level of effort chosen by an agent with relative deprivation compared to the level chosen by an agent without relative deprivation (with $e_{Leq}^{**} \leq \bar{E}$).

"Positional stimulated agent": When $G_e(.) > 0$ and $G_e(.) < -\theta\beta_M\Delta y G_{y^R}(.)$ (Condition II), relative deprivation increases the optimal level of effort chosen by an agent with relative deprivation compared to the level chosen by an agent without relative deprivation (with $e_{Leq}^{**} \leq \bar{E}$).

"Positional discouraged agent": When $G_e(.) > 0$ and $G_e(.) > -\theta\beta_M\Delta y G_{y^R}(.)$ (conditions III), relative deprivation decreases the optimal level of effort chosen by an agent compared to the level chosen by an identical agent without relative deprivation (with $e_{Leq}^{**} \leq \bar{E}$)..

Proof. direct from equations 3.11 and the functional form of $G(y_i^R, e_i)$.

When $G_e(.) < 0$, the equilibrium effort level e_{Leq}^* is always lower than e_{Leq}^{**}. Under condition I, given the same level of effort, those agents with relative deprivation perceive a lower cost for additional relative effort, and therefore, they make a higher effort. In this case, function $G_e(.)$ can be interpreted as implying that agents get utility from relative effort. As such, it is not surprising that more effort is the outcome (self-motivated effect).[18]

Meanwhile, when $G_e(.) > 0$, the reasoning is somewhat different, because relative effort is always a cost. Namely, given an expected income gap with respect reference group, relative deprivation generates lower utility among those agents who have made a greater effort. However, if $G_e(.) < -\theta\beta_M\Delta y G_{y^R}(.)$ the higher disutility of high relative effort is compensated by a lower relative income gap. In this case, a high relative deprivation increases marginal utility and it mitigates the additional marginal cost of effort. Therefore, the encouraged effect dominates because there is high opportunity for income mobility ("relative effort pays" because $\theta\beta_M\Delta y$ is high). However, if $G_e(.) > -\theta\beta_M\Delta y G_{y^R}(.)$, then e_{Leq}^* is higher than e_{Leq}^{**}. In this case, when agents suffer high relative deprivation, they perceive a higher cost for additional relative effort. Therefore, the marginal utility of a reduction in the relative deprivation is lower than

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[18]Kandel and Lazear (1992) use similar argument to explore how peer pressure operates on worker effort. They suggest that the peer pressure function can be interpreted as implying that workers get utility from effort.

the marginal disutility of a higher effort in the relative component. The reduction of relative deprivation is more demanding in terms of effort and agents are discouraged.

We can now consider the effects of an exogenous increase in reference group income among agents who care about their relative deprivation $(E(y^R < 0)$. By differentiating the individual's first order condition for the choice of effort we find the following expression:

$$de_{Ueq}^{**}/dy^{RG} = \frac{\alpha a \left[\theta \beta_M \Delta y G_{y^R y^R}(.) + G_{y^R e}(.)\right]}{1 + \alpha a G_{ee}(.) + \alpha a(\theta \beta_M \Delta y)^2 G_{y^R y^R}(.) + 2\alpha a \theta \beta_M \Delta y G_{y^R e}(.)} \quad (3.13)$$

The expression in the numerator of Eq. 3.13 determines the sign of de_{Ueq}^{**}/dy^{RG} (the denominator is positive due to the second order condition). First, note that $G_{y^R y^R}(.)$ is positive, therefore, the sign of this expression depends on the sign of $G_{y^R e}(.)$. What can we say about the sign of $G_{y^R e}(.)$?. When the inverse of effort and relative income are complements in the relative deprivation term, the sign of $G_{y^R e}(.)$ is positive.[19] Under this condition, a higher income gap leads to higher effort. If they are not complements, the sign of de_{Ueq}^{**}/dy^{RG} is ambiguous, and it depends on the magnitude of $\theta \beta_M \Delta y G_{y^R y^R}(.)$, namely, the sign depends on relative rewards and ex-ante inequality. We can express this ideas in more formal way :

> Proposition 3: When $E(y_i \leq y^{RG})$, under non additive comparisons and asymmetry in the income comparison:
>
> "Income gap self - encouraged agent'''": When $G_{y^R e}(.) > 0$ (Condition IV), higher reference income always leads to additional effort ($de_{Leq}^{**}/dy^{RG} > 0$ with $e_{Leq}^{**} \leq \bar{E}$).
>
> "Income gap - stimulated agent": When, $G_{y^R e}(.) < 0$ and $\theta \beta_M \Delta y G_{y^R y^R}(.) > -G_{y^R e}(.)$ (Condition V), higher reference income always leads to additional effort ($de_{Leq}^{**}/dy^{RG} > 0$ with $e_{Leq}^{**} \leq \bar{E}$).
>
> "Income gap - discouraged agent": When, $G_{y^R e}(.) < 0$ and $\theta \beta_M \Delta y G_{y^R y^R}(.) < -G_{y^R e}(.)$ (Condition VI), higher reference income always leads to lower effort ($de_{Leq}^{**}/dy^{RG} < 0$ with $0 \leq e_{Leq}^{**} \leq \bar{E}$).

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[19]These ideas are used in Bowles and Parker (2005) to discuss the importance of "Veblen effect" in the individual's allocation of time between labor and leisure. Dalton, Ghosal and Mani, (2014) assume a similar assumption to incorporate income aspiration on utility function.

"Indifferent agents": When $G_{y^R e}(.) < 0$ and $\theta \beta_M \Delta y G_{y^R y^R}(.) = -G_{y^R e}(.)$ (Condition VII), individuals do not respond to changes on reference group income $(de^{**}_{Ueq}/dy^{RG} = 0)$.

Firstly, we observe that the reference groups composition is relevant in explaining effort levels. The relationship between effort and P depends on whether leisure and relative income are complements. Furthermore, the intensity of the relative deprivation effect on effort decision is higher when expected differences between individuals with different social origins are higher.[20]

Secondly, conditions IV or V establish a positive relation between effort and reference income. However, there is a difference between them. In the former, higher reference group income decreases the marginal cost of relative effort (relative effort generates utility). As a result, a higher reference group income can increase the effort levels through two channels, the higher marginal utility of relative income and the lower marginal cost of relative effort. In the second case, a higher relative deprivation induces a higher marginal cost of relative effort (relative effort represents a cost). However, this effect is dominated by the higher marginal utility of the relative income gap. On the contrary, condition VI establishes a negative relation between effort and reference group income $(de^{**}_{Ueq}/dy^{RG} < 0)$. In this case, a higher relative deprivation increases the marginal utility of $G(.)$, but this effect is dominated by an increase in the marginal disutility of relative effort, causing a reduction in the levels of effort. As a result, a higher relative income gap reduces the effort level.

Thirdly, under conditions V and VI, relative effort represents a cost, in accordance with standard economic models, but agent's effort response is ambiguous. In this case, the parameters of economic inequality are more relevant in explaining the agent's effort response. Therefore, when returns of effort and ability (θ), expected ability (β), and income premium are higher $(\triangle y)$, the feasibility income gap - discouraged agent is lower. We will discuss this issue in the next section.

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[20]Note that differentiating the individual's first order condition for the choice of effort with respect P we find the following expression: $de^{**}_{Ueq}/dP = \frac{-\alpha a \Delta y (e^b_L - e^b_U - \Delta \pi) \left[\theta \beta_M \Delta y G_{y^R y^R}(.) + G_{y^R e}(.) \right]}{1 + \alpha a G_{ee}(.) + \alpha a (\theta \beta_M \Delta y)^2 G_{y^R y^R}(.) + 2\alpha a \theta \beta_M \Delta y G_{y^R e}(.)}$. Observe that $-\alpha a \Delta y (e^b_L - e^b_U - \Delta \pi) > 0$.

The role of effort rewards and ex-ante inequality on relative deprivation and attitude toward effort

If we assume that relative effort represents a cost ($G_e(.) > 0$), we are able to examine how the magnitude of effort rewards encourages (or discourages) agents. Observe that condition VI defines the locus where individuals face relative deprivation, but they do not respond to a change in reference group income ($de^{**}_{Ueq}/dy^{RG} = 0$). An interesting analytical result stems from differentiating condition VI with respect to θ [21]. This allow us to identify the region of "indifferent agents", and thus stimulates and discouraged agents.

The locus which defines "indifferent agents" depends essentially on the sign of $G_{y^R y^R y^R}(.)$. When $G_{y^R y^R y^R}(.) < 0$ (Principle of diminishing transfers), there is a function $f(G(.), \theta, e_{Leq}, \triangle \pi)$: $\tilde{\theta}(\theta) = d\theta G_{y^R y^R y^R}(.) = -\theta G_{y^R y^R y^R}(.) dy^R/d\theta$, which defines the condition that must be met for $de^{**}_{Ueq}/dy^{RG} = 0$) for alternatives values of parameter θ. [22] Therefore, given $G(.)$ and $\triangle \pi$, $de^{**}_{Ueq}/dy^{RG} < 0$ if $\theta^{Low} < \tilde{\theta}(\theta)$ and $de^{**}_{Ueq}/dy^{RG} > 0$ if $\theta^{high} > \tilde{\theta}(\theta)$. Observe that there is no monotonous relationship between the sign of de^{**}_{Ueq}/dy^{RG} and θ. How agents respond to an increase in θ depends on $G_{y^R y^R y^R}(.)$ and $G_{y^R y^R}(.)$. The increase of θ has a direct positive effect on effort, because it improves expected relative deprivation ($dy^R/d\theta > 0$). However the higher relative income decreases the sensibility of the marginal utility of relative deprivation ($\downarrow G_{y^R y^R}(.)$ because $G_{y^R y^R y^R}(.) < 0$), which reduces the incentive to increase effort (utility gains are higher when y^R is low). Given these effects in opposite directions, it is ambiguous how individuals responds to higher θ (See Figure A1 in the Annex I).

However, this ambiguity disappears when $G_{y^R y^R y^R}(.) > 0$, because a function $\tilde{\theta}(\theta)$ does not exist. In this case, given the function $G(.)$ and $\Delta \pi$, only one value of θ, $\tilde{\theta}$, meets ($de^{**}_{Ueq}/dy^{RG} = 0$). Therefore, regardless of the functional form of $G_{y^R y^R}(.)$ and $G_{y^R y^R y^R}(.)$, $de^{**}_{Ueq}/dy^{RG} < 0$ if $\theta < \tilde{\theta}$ and $de^{**}_{Ueq}/dy^{RG} > 0$ if $\theta > \tilde{\theta}$. In this case, an increase of θ increases expected relative deprivation and the sensibility of marginal utility of relative deprivation. Both effects play in the same direction, and effort will increase.

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[21] To simplify, we assume that G_{ey^R} is constant and $e^b_{Ueq} < e^b_{Leq}$

[22] Under the Principle of diminishing transfers, given two identical individuals, i and j, who only differ in their expected absolute income level ($E(y_i < y_j)$, the same reduction in relative income gap, generates a higher increase in an individual's utility for i than for j. Namely, the same reduction in relative deprivation causes a higher increase in utility when an individual earns 1000, than when he earns 10000. This principle is not true when $G_{y^R y^R y^R}(.) > 0$ (Kolm, 1975).

In the latter case, it is useful to examine the relationship between θ and $\Delta\pi$ (effort rewards and ex-ante inequality rewards) when $de^{**}_{Ueq}/dy^{RG} = 0$. There is a function $f(G(.), \theta, e_{Leq}, \triangle\pi) : \tilde{f}(\theta, \triangle\pi)$, which defines the set of all values of θ and $\triangle\pi$ where individuals do not respond to changes in reference group incomes. Given previous assumptions, we can conclude that $\frac{\tilde{f}_\theta(\theta, \triangle\pi)}{\tilde{f}_{\triangle\pi}(\theta, \triangle\pi)} > 0$. In this case, a higher θ generates incentives to increase effort, which can be compensated with a higher $\triangle\pi$. To make this result a little more concrete, assume two economies A and B, with $\tilde{f}_A(\theta_A, \triangle\pi_A)$ $= \tilde{f}_B(\theta_B, \triangle\pi_B)$, but the former presents higher ex-ante inequality ($\triangle\pi_A > \triangle\pi_B$). In order for there to be an stimulated income gap effect on effort decision, the economy A will require a higher effort reward levels θ such $\theta > \theta_A > \theta_B$.[23]

3.6   An analysis of efficiency and income mobility

It is useful to consider the properties of an equilibrium in which many effort decision-makers act as the presented model. Economic outcomes are then appropriately thought of as a Cournot-Nash equilibrium, so that, each agent takes other's choices as given. Assume a continuum of agents with origin I_L, differentiated by composition of reference group (P_i). We retain the assumption A.VII*3.3* ($P_i = 1$ for the agents with origin I_U) and, to simplify, henceforth we assume $e^b_U \leq (1-\alpha)a\theta\beta_M\Delta y$ (A.VIII), and therefore, for agents with upper-class social origins $E(y^R \mid I_U) \geq 0$. As the expected income of agents with I_U affects the utility of agents with origin I_L, but the inverse is not true, the former could be interpreted as leaders and the seconds as followers (Clark and Oswald;1998). Under these conditions, the expected social welfare is given by:

$$
\begin{aligned}
W = E(U \mid I_U) + n \int E(U \mid I_L) = \\
(1-\alpha)\Delta y(\theta\beta_M e_U + \Delta\pi + \pi) - \frac{e_U^2}{2a} + \\
n \int \left[(1-\alpha)\Delta y(\theta\beta_M e_L(P) + \Delta\pi + \pi) - \alpha G(y^R(P), e_L(P)) - \frac{e_L(P)^2}{2a} \right] dp
\end{aligned}
\tag{3.14}
$$

where the number of agents with origin I_U were normalized to unity, and $n > 0$ represents the number of agents I_L for each agent with origin I_U. Under perfect information the expected effort are $e^b_{Ueq} = e_{Ueq}$ and $e^b_{Leq} = \int p e_{Leq}(P) dp$.

For society to be at an optimum:

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[23]The sign of $\frac{\tilde{f}_\theta(\theta, \triangle\pi)}{\tilde{f}_{\triangle\pi}(\theta, \triangle\pi)}$ is indeterminate when $G_{y^R y^R y^R}(.) < 0$.

$$e_{Ueqopt} = [(1-\alpha)\theta\beta_M\Delta y - \lambda_1]a \tag{3.15}$$

$$e_{Leqopt}(P) = a\left[(1-\alpha)\theta\beta_M\Delta y - \alpha\theta\beta_M\Delta y G_{y^R}(y^R(P), e_L(P)) - \alpha G_e(y^R(P), e_L(P)) - \frac{\lambda_2}{s}\right] \tag{3.16}$$

$$\lambda_1 = -s\int\left[f(P)(1-P)\alpha\theta\beta_M\Delta y G_{y^R}(y^R(P), e_L(P))dp\right] \tag{3.17}$$

$$\lambda_2 = -s\int\left[f(P)P\alpha\theta\beta_M\Delta y G_{y^R}(y^R(P), e_L(P))dp\right] \tag{3.18}$$

$$\int pe_{Ueqopt}(P))dp - e^b_{Ueqopt} = 0 \tag{3.19}$$

$$\int pe_{Leqopt}(P))dp - e^b_{Leqopt} = 0 \tag{3.20}$$

where λ_1 and λ_2 are the multipliers on constrains (3.19) and (3.20) respectively. Therefore, if we compare equations 3.16 and 3.15 with the previous equations 3.7 and 3.7.b for private effort choices, expected equilibria are not optimal. By concavity of (3.14) and due to λ_1 and λ_2 being positive (from Eq. 3.17 and 3.18), socially expected desirable levels of effort are below those which agents individually make. This is because effort decisions affect the relative deprivation of others and because of the well-known 'rat-race' effect induced by the status motive. Because agents ignore the externalities which their decisions generate, the equilibrium based on individual decisions will be sub-optimal. This result is in accordance with the findings of economic models in which individual utility depends on relative situation (Clark and Oswald, 1998; Piketty, 1998; Frank,1997; 2005). However, these derivations allow us to distinguish two possible sources of externalities. On one hand, Eq. 3.15 demonstrates that the effort of agents with origin I_U (leaders) generates a negative externality on the decisions of agents with origin I_L(followers). Furthermore, this externality "between" social origins, will be higher when $\Delta\pi$ is higher. As a result, regardless of the effort decision of agents with origin I_L, a lower ex-ante inequality reduces expected ineffi-

ciency (lower ex-ante inequality reduces inefficiency). [24]. On the other hand, there is a "within externality", which comes from the effort decisions of the peer with origin I_L. Finally, note that the source of inefficiency is that agents make too much effort. Obviously, inefficiency will be lower, when unstimulated effect plays a role, but in this case, the expected upward mobility will be lower for agents with origin I_L.

4 A MODEL OF EFFORT CHOICE WITH REFERENCE GROUP AND INTERTEMPORAL LEARNING

The results of the previous section could be interpreted as a benchmark, which considers a situation in which there is perfect information (expected effort is known, constant and exogenous) or a long-run equilibria with completed forward-looking agents. Now we assume that agents with origin I_L do no know the effort of the peers of their generation and they choose their effort based on their belief (e_L^b). Each generation updates their belief with respect to previous generation belief by trial and error methods using local knowledge based on their peer past experience. Beliefs are updated by a backward-looking learning process, that is, in light of the recent experience of peers with the same social origin from a previous generation. This establishes a connection between expected effort and performance in terms of the income mobility of a previous generation. Bowles (2004) argues that backward-looking learning approach has advantages when compared to the forward-looking learning process.[25] We assume that agents incorporate information of the economic performance of the previous generation when they update their *apriori* public beliefs, which are transmitted from previous generations.[26] Finally, this learning procedure seems useful to explaining the forma-

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[24]Observe that equilibrium based on individual decisions will be optimum when P=0. But in this case, effort decision reduces income mobility. Furthermore, aggregate inefficiency will be higher when more agents with origin I_L present high reference income ($F(P)' > 0$).

[25]Bowles (2004) considers backward-looking learning process inside evolutionary game theory. In contrast to the forward-looking agents in classical game theory, this approach addresses the history of the agents

[26]Other papers have used this learning procedure and they place on emphasis on the information transmission between generations and the significance of past trajectories in order to explain heterogeneous beliefs equilibrium. Piketty (1995) used Bayesian learning to update the belief about the parameters of the economy, Piketty (1998) to explain the public beliefs about status, and Breen and García – Peñalosa (2002) to describe the difference in preferences across genders.

tion of aspirations based on social interactions, where individual economic aspiration is conditioned by the experiences of other agents in their cognitive neighborhood (Appadurai, 2004, Genicot and Ray, 2010).

In this section we focused on agents with a low social background as the best case study. The assumptions of the previous section establish a dynamic leader-follower between agents with origin I_L and agents with origin I_U. Therefore, in order to analyze the role of the reference group as a determinant of income inequality persistence, we can retain assumption A.VIII ($\bar{e}_U^b \leq (1-\alpha)a\theta\beta_M\Delta y$ and is exogenous) because agents with origin I_L can't affect the effort decision of agents from I_U. This implies that the optimal effort of agent with origin I_U is $e_{Ueq} = (1-\alpha)a\theta\beta_M\Delta y$, which represents a benchmark for agents with origin I_L. In the remainder of this section, we focus on agent with origin I_L (for notational simplicity we omit the social origin sub-index L and U for the rest of this section).

4.1 The information structure

We assume that agents are uncertain about the real peers' effort when they choose their effort level. Each agent takes others' effort as given within the same period, but they update their beliefs about e^b between generations. Informational assumptions A.I, A.II and A.III from section 3 remain the same. Individual effort levels are not publicly observable, but agents know that they are between a certain "high effort level" ($\bar{e} \leq \bar{E}$) and a certain low effort level" ($\underline{e} \geq 0$), with ($\bar{e} \geq \underline{e}$) (A.IX). The current generation know the social mobility experienced by the previous generation which represents a signal of their effort levels (A.X). Public beliefs about effort are transmitted across generations, therefore generation $t+1^{th}$ has a *priori* information based on real beliefs of generation t^{th}(A.XI).

Given assumption A.IX, the expected effort for their peers in generation t^{th} is defined as:

$$e_t^b = \mu_t\bar{e} + (1-\mu_t)\underline{e} \tag{4.1}$$

where μ_t is the public belief of generation t about the participation of high effort agents among economically successful agents from the previous generation with origin I_L.This parameter could be interpreted as the subjective probability attached by

the entire generation that $\bar{e}$ was the effort of agents with origin I_L (Prospect theory suggests that agents weigh their options based on subjective distribution function).

For each agent with origin I_L there is a latent variable which describes the relation between economic success and effort, which is defined in equation (3.3) as $Y'_{it} = \pi + \theta \beta e_{it}$. An agent i from generation t does not observe e_{it}, but he know individual social mobility trajectories (y_1 or y_0) of all agents from generation $t - 1^{th}$(A.X). For this reason, for the belief of generation $t + 1$, the mobility outcome of agents I_L from generation t represents a signals about the effort of agent with origin I_L.

It is useful to consider that the economic performance is stochastically related to effort, incorporating a random variable v_{it}. Therefore, the expected probability that n agents with origin I_L from generation t reaching y_1 is defined as:

$$E(Pr(y_{1t} = y_1, y_{2t} = y_1, \ldots y_{nt} = y_1 \mid i = 1 \ldots n \in I_L)) = \prod_{\forall i \in I} (\pi + \theta \beta_M e_{it} + v_{it}) \quad (4.2)$$

where v_{it} represents an idiosyncratic shock (which reflects income realization) for each generation t and agent i, with $E(v_{it}) = 0$ and $0 \leq \pi + \theta \beta e_{it} + v_{it} \leq 1$, for $0 \leq e_{it} \leq E$. Taking x_t as the real share of successful agents with origin I_L from generation t, agents can derive the probability of the signal $x_t = x'_t$, conditional on the state being v'_t:

$$Pr(x_t = x') = \Omega(\varepsilon'_t, v'_t \mid v'_t) = \alpha(e_{M_t}(\varepsilon'_t), v'_t \mid v'_t) \quad (4.3)$$

where ε_t and v_t are vectors of n dimension, which respectively reflect individual efforts in t $(e_{1t}, e_{2t} \ldots, e_{nt})$ and n random variables $(v_{1t}, v_{2t} \ldots, v_{nt})$, and ε'_t and v'_t are particular realizations of both vectors. For notational simplicity, we introduce the function $\alpha(.)$, whose argument is the mean effort of agents with origin I_L in t (e_{M_t}), which is a linear function of each element in the vector ε_t. As agents know π, θ, β_M, Δy and Δy, given e_{M_t} they know the distribution of signals (A.I, A.II and A.IX), which describes the expected share of successful agents with origin I_L from generation t, conditional on the state v'_t.

4.2 Intergenerational learning

Agents with origin I_L from generation $t+1$ know the real percentage of economically successful agents with origin I_L in the previous period (x_t), but they do not observe which of them made a high effort (A.X). By assumption A.XI previous generation transmitted their beliefs, therefore, current generation have a *priori* beliefs $e_{t+1}^{apriori}$ about $e_{M_{t+1}}$, which is defined as: $e_{t+1}^{apriori} = e_t^b = \mu_t \bar{e} + (1-\mu_t)\underline{e}$ (Note $\mu_{t+1}^{apriori} = \mu_t$). Each generation $t+1$ observes a signal x_t', which is received from the predecessor generation. Since mobility performance is only stochastically related to effort, "evaluation errors may occur". If $x`_t \neq \alpha(e_t^{apriori}, v_t' \mid v_t')$ there is an error on a *priori* beliefs . As a result, even if agents do not know the latent variable Y_t', based on the signals x_t' generation $t+1$ could update their priori beliefs about the effort of their peers (and their effectiveness for economic success) according to Bayes' rule.

Observe that the importance of those errors depends on the correlation between e_{it} and v_{it}. On one hand, when $\bar{\sigma} = Corr(e_{it}, v_{it}) > 0$ the shock does not "redistribute" economically successful agents between low and high effort agents. As a result, the "effort pays" and high effort agents dominate between successful agents. On the other hand, an alternative hypothesis is that $\underline{\sigma} = Corr(e_{it}, v_{it}) < 0$, in which case the shock "redistributes agents", namely some agents with low effort achieve economic success. In this second case, although effort has positive impact on the probability of high income, the effort reward is relatively lower compared to the first case. As a result, the proportion of low effort level is relatively high among economically successful agents, and then e_t^b (and μ_t) should be lower. Observe that the sign of this correlation represents two states of the world.[27]

The distribution function of signals depends on the real state of the world. The probability that the public signal x_t' is realized conditional on the state being $\bar{\sigma}$ or $\underline{\sigma}$ are defined as:

$$Pr(x_t = x') = \alpha(e_{M_t}(\varepsilon_t'), v_t' \mid \bar{\sigma}, h_{t-1}) = \overline{\alpha}(e_{M_t}(\varepsilon_t'), v_t' \mid h_{t-1}) \qquad (4.4)$$

$$Pr(x_t = x') = \alpha(e_{M_t}(\varepsilon_t'), v_t' \mid \underline{\sigma}, h_{t-1}) = \underline{\alpha}(e_{M_t}(\varepsilon_t'), v_t' \mid h_{t-1}) \quad (4.4b)$$

---

[27]Correlation could be interpreted as an expression of the heterogeneous capacity of the agents to respond to different shocks, given their effort.

where h_{t-1} describes the decisions history of all agents I_L from previous generations $(t-1, t-2, ...)$.

As $\mu_{t+1}^{apriori}$ is an a *priori* probability (subjective weight) assigned to high effort $\bar{e}$, it also represents the subjective probability attached by a generation $t+1$ that $\bar{\sigma}$ is the true state of the world. Following Piketty (1995, 1998) and Breen and García – Peñalosa (2002), we assume that intergenerational learning takes the form of Bayesian updating, with beliefs being updated by the current generation from the previous generations. An individual from generation $t+1^{th}$ uses mobility results to update their a *priori* beliefs.

The sequence of events is as follows. The agents with origin I_L from generation t base their effort decisions on their beliefs about the expected effort of their peer in the current generation (e_t^b). They choose their effort levels and, after the realization of v_t, they obtain y_0 or y_1 (they generate the public signal x_t'). The belief of generation t (e_t^b) is inherited by the next generation $(e_{t+1}^{apriori} = e_t^b$ and $\mu_{t+1}^{apriori} = \mu_t)$. The updated belief of generation $t+1$ (e_{t+1}^b) combines that a *priori* information with the mobility outcome of generation t. After the output of mobility income of generation t is realized, the next generation updates their a *priori* beliefs and they choose their effort level based on their updated beliefs. Bayesian learning implies that the outcomes of the previous generation are interpreted in the light of the a *priori* beliefs. As a result, an effort belief (e_{t+1}^b) combines a *priori* information transmitted from previous generations e_t^b and information about the mobility experienced by the previous generation x_t'. The posterior beliefs of the following generation which observe the signal x_t' is given by Bayes' rule:

$$\mu_{t+1} = \frac{Pr(\bar{\sigma} \cap x_t' | h_{t-1})}{Pr(x_t' | h_{t-1})} = \frac{\frac{Pr(\bar{\sigma}|h_{t-1}).Pr(x_t = x_t'|\bar{\sigma}, h_{t-1})}{Pr(\bar{\sigma}|h_{t-1})Pr(x_t=x_t'|\bar{\sigma}, h_{t-1}) + (1-Pr(\bar{\sigma}|h_{t-1}))Pr(x_t=x_t'|\underline{\sigma}, h_{t-1})}}{\frac{\mu_t Pr(x_t=x_t'|\bar{\sigma}, h_t)}{\mu_t Pr(x_t=x_t'|\bar{\sigma}, h_{t-1}) + (1-\mu_t)Pr(x_t=x_t'|\underline{\sigma}, h_{t-1})}} \quad (4.5)$$

where the *priori* belief $\mu_{t+1}^{apriori}$ is equal to μ_t, and the terms $Pr(x_t = x_t' \mid \bar{\sigma}, h_{t-1})$ represents the conditional probability of the public signals x_t' given occurs h_{t-1} and that the true state is $\bar{\sigma}$. These probabilities were defined when we introduced the distribution function of signals (Eq. (4.4) and (4.4)b). Agents known the functions of distribution of signals, then replacing in Eq.(4.5) we arrive at the following expression:

$$\mu_{t+1} = \frac{\mu_t \bar{\alpha}(e_t^b, v_t' | h_{t-1})}{\mu_t \bar{\alpha}(e_t^b, v_t' | h_{t-1}) + (1-\mu_t)\underline{\alpha}(e_t^b, v_t' | h_{t-1})} \quad (4.5b)$$

This function describes the evolution of generation's beliefs over time. Note that this function depends on a priori beliefs, as a result, the same mobility outcome can give rise to different posterior beliefs. If we consider equations(4.1) and 4.5b together, the effort beliefs are updated according the flowing rule:

$$
\begin{cases}
\overline{\alpha}(e_t^b, v_t' \mid h_{t-1}) > \underline{\alpha}(e_t^b, v_t' \mid h_{t-1}) & \iff \mu_{t+1} > \mu_t \iff e_{t+1}^b > e_t^b \\
\overline{\alpha}(e_t^b, v_t' \mid h_{t-1}) < \underline{\alpha}(e_t^b, v_t' \mid h_{t-1}) & \iff \mu_{t+1} < \mu_t \iff e_{t+1}^b < e_t^b
\end{cases}
\tag{4.6}
$$

Whether the updated weight placed on $\bar{e}$ is greater than the *prior* probability depends on whether, for the level of effort chosen by the previous generation, the signal observed is more likely to have occurred for $\bar{\sigma}$ than for $\underline{\sigma}$. If a generation t^{th} experienced a relatively high mobility outcome with respect to his *priori* beliefs, the conditional probability of this event given previous history h_{t-1}, is greater for $\bar{\sigma}$ than for $\underline{\sigma}$. As such generation $t+1$ places a higher weight on $\bar{e}$. The opposite holds for the case of low mobility results. The rationality of the updating belief rule is the following: when agents of generation $t+1$ have an a *priori* belief that their peers had made a high effort but were not rewarded with upward mobility, there will be some downward adjustment of the expected effort for their current peers. For example, if $x_{t+1}^{apriori}$ is the a *priori* belief in generation $t+1^{th}$ about expected successful agents with origin I_L and if x_t' is the realization in period t, the updating rule for beliefs determine $e_t^b > e_{t+1}^b$ when $x_{t+1}^{apriori} > x_t'$. In contrast, high achievable performance in the previous generation should induce rational agents to expect higher effort in their next generation peers, namely $e_t^b < e_{t+1}^b$ if $x_t^{apriori} < x_t'$.

A general property of this form of Bayesian learning is that the stochastic process μ_t describes a martingale: what generation t expects its successors to know next period is exactly what generation t knows today. Namely, the agent's best guess in generation $t+1$, as to his posterior in any later period is his posterior beliefs in period t, namely $E(\mu_{t+m} \mid \mu_t, h_t)$ with $m > 1$ (Aghion et al., 1991; Piketty, 1995; Smith and Sørensen, 2000). As a result, $E(e_{t+m}^b \mid \mu_t, h_{t-1}) = E(\mu_{t+m}^b \mid \mu_t, h_{t-1})\bar{e} + (1 - E(\mu_{t+m}^b \mid \mu_t, h_{t-1}))\underline{e} = e_t^b$. Assume, without loss of generality, that the true state of the world s $\bar{\sigma}$(namely "effort always pays"). [28] Therefore $\mu_t = 1$ is equivalent to allocating full weight on the truth. Pick $\overline{\sigma} \neq \underline{\sigma}$, with $\mu(e_{t-1}^b, \overline{\sigma}, v_t) > 0$, and define for any $t > 1$ the

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[28]Piketty (1995) discusses extensively the reasons that justify that assumption.

likelihood $I_t = \frac{\mu(e^b_{t-1}, \overline{\sigma}, v_t)}{\mu(e^b_{t-1}, \underline{\sigma}, v_t)}$, follows a stochastic process $\{\mu_t\}$, which describes a martingale conditional on the true state of the world. As a result, standard martingale convergence results can be applied (Aghion et al., 1991; Piketty, 1995, Smith and SØrensen, 2000 and Breen and García – Peñalosa, 2000). Piketty (1995) and Breen and García–Peñalosa (2000) derived three propositions about this process, which could be interpreted in terms of our learning process.

First, the martingale convergence theorem implies that the likelihood ratio, and hence beliefs, converge in the long term. For any initial beliefs, μ_0, in the long term beliefs converge toward some stationary beliefs, μ_∞ with probability one. Therefore, there is a stable solution about the level of expected effort, which is defined as: $e^b_\infty = \mu_\infty \overline{e} + (1 - \mu_\infty)\underline{e}$. Second, given the true state of the world $\overline{\sigma}$, the Bayesian updating function defined in Eq.4.5 has three fixed points. One of them is not stable $\mu_{1\infty} = 0$. There are two stable long term equilibrium beliefs, one is an interior fixed point $\mu_{2\infty} > 0$ and the other is a corner solution $\mu_{3\infty} = 1$. As a result, both stationary beliefs allocate a positive weight to the true state of the world. In terms of effort beliefs, $e^b_\infty = \underline{e}$ is not a stable solution, meanwhile, $e^b_{2\infty} = \mu_{2\infty}\overline{e} + (1 - \mu_{2\infty})\underline{e}$ and $e^b_{3\infty} = \overline{e}$ are stable solutions. Finally the interior solution $\mu_{2\infty}$ holds:

$$\overline{\alpha}(e^b_t(\mu_{2\infty}), v'_t \mid h_{t-1}) = \underline{\alpha}(e^b_t(\mu_{2\infty}), v'_t \mid h_{t-1}) \Longleftrightarrow \mu_{t+1} = \mu_t \Longleftrightarrow e^b_{t+1} = e^b_t \quad (4.7)$$

this expression implies that when agents hold the *prior* belief $\mu_{2\infty}$, the resulting expected probability is the same under $\overline{\sigma}$ or $\underline{\sigma}$. If initial beliefs are $\mu_0 < \mu_{2\infty}$, then it converges to $\mu_{2\infty}$ with a probability of one. As a result, $e^b_{2\infty} = \mu_{2\infty}\overline{e} + (1 - \mu_{2\infty})\underline{e}$. In contrast, if the initial beliefs are higher than $\mu_{2\infty}$, then they will be attracted with positive probability, $Pr(\mu, \mu_{2\infty})$, by $\mu_{3\infty} = 1$, and with positive probability $1 - Pr(\mu, \mu_{2\infty})$, by $\mu_{2\infty}$. Breen and García-Peñalosa (2002) named $\mu_{2\infty}$ as "confounded learning beliefs". At this point nothing can be learned from the previous generations' signals, and a *priori* beliefs are equal to the posterior. They demonstrated that the probability of converging to the true belief is given by:

$$Pr(\mu, \mu_{2\infty}) = \frac{\mu_0 - \mu_{2\infty}}{\mu_0(1 - \mu_{2\infty})} \quad (4.8)$$

As a result, the long term equilibrium belief depends on the initial beliefs and the

Figure 4.1: The equilibrium of beliefs
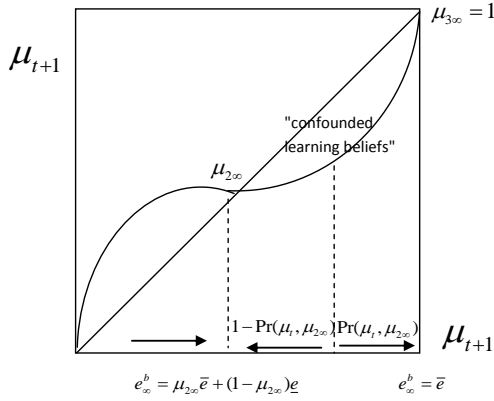
quality of the public signal information. This result is due to the fact that the same mobility outcome can give rise to different posterior beliefs depending on the probabilities initially attributed to each situation. Successive learning across generations may be complete, as a result, generations will access the true value of σ, $\overline{\sigma}$. In this case, an equilibrium belief about the expected effort of an agent with origin I_L is $e_{3\infty}^b = \overline{e}$. Namely, in this case "effort always pays" in the long term, and agents with origin I_L expect their peers to excert a high level of effort. One point worth noting here is that $\overline{e}$ may not be the "true" mean effort. This expected level of effort is the most likely value given $\overline{\sigma}$ is the true state of the world, h_{t-1} the history of generations with social origins I_L and μ_0 the initial beliefs. In other terms, evidence shows that effort pays, and that successive learning across generations leads to the highest expected effort. However, the learning process across generations may be incomplete, in this case agents perceive that effort rewards are relatively low, even if this is not true. As a result, agents place a strictly positive weight on the true state of the world ($\overline{\sigma}$), and long term equilibrium of the expected effort $e_{2\infty}^b$ is lower than $\overline{e}$, but is higher than $\underline{e}$. Although "effort pays" and promotes high income mobility, initial beliefs and mobility trajectories lead in the long term, to relative lower expected effort for agents with origin I_L.

4.3 An equilibrium analysis with intergenerational learning with self-motivated agents

In section 3 we characterized different individual responses to the relative income effect depending on agent characteristics and circumstances. We interpret these characteristics to be part of the personality of the agent and they are explained by idiosyncratic term. To simplify, in this section we assume that all agents are identical and they are self-motivated agents (Conditions I and V). Even though this assumption simplifies the analysis, it is worth noting that in this scenario, the relative income effect always motivates a high effort. Therefore, it allows us to explore how an agent's effort decision is affected by income mobility results and expected relative deprivation.

Under imperfect information, the relative deprivation with respect reference group income establishes a relationship between generation in two ways. On one hand, the probability of economic success depends on social origin. On the other hand, there is an indirect channel, because the experience of previous generations affects the beliefs about expected effort, and they determine the incidence of reference groups through their expected relative deprivation. Equations 4.6 provides a rational updating process, where society learns from the mobility outcome. The Bayesian learning mechanism implies that history is important in determining equilibrium beliefs and public expected effort and therefore the reference group income level.

In steady state agents with origin I_L and the same P will choose the same optimal effort $e_{L\infty}(P_i)$ and it is constant: $e_{Leqt-1}(P_i) = e_{Leqt}(P_i)$ and $e_{t-1}^b = e_t^b$. In steady state $e_L^b = e_{L\infty}^b$, considering $E(y_\infty^R \mid I_L) = \Phi(e_{Leq}(P), e_{L\infty}^b, \overline{e_U^b}, P)$ in Eq. 3.11, we arrive at the following expression:

$$
e_{L\infty}(P_i) = \begin{cases} e_{L\infty}^* = (1-\alpha)a\theta\beta_M\Delta y & if\, E(y_\infty^R \mid I_L) \geq 0 \\ e_{L\infty}^{**} = e_{L\infty}^* - a\theta\beta_M\Delta y G_{y_\infty^R}() - a\alpha G_e() & if\, E(y_\infty^R \mid I_L) < 0\, \&\, e_{L\infty}^{**} < E \\ e_{L\infty} = E & if\, e_{L\infty}^{**} \geq E \end{cases}
$$

(4.9)

In the steady state, the Bayesian learning function leads to social beliefs $e_{L\infty}^b$. As a result for self-stimulated agent with origin I_L the models predict two possible scenarios about effort level in the steady state. First, when $\mu_0 > \mu_{2\infty}$ agents beliefs will be attracted with probability $Pr(\mu, \mu_{2\infty})$ by $e_{L\infty}^b = \overline{e}$. In this case, agents with higher P choose a high effort level because their relative deprivation and reference income are

relatively high. They are stimulated by the expected income of agents with origin I_U but also by their peers with origin I_L. When P is relatively low (agents with origin I_L compare with their peers), they tend to choose higher effort level, because expected effort for agents with origin I_L is high. In this case, the expectation of peer's effort will increase, and so will individual effort for the future. The intensity of this effect is higher, among agents whose reference group composition has low P. However, the effort in steady state will be always equal or higher for those agents with higher P, because they include more agents with origin I_U in their reference group. When $P > 0$, reference group promotes higher income mobility, meanwhile, when P=0 result are consistent with "self fulfilling belief" of Piketty's model.

On other hand, due to the initial condition and the past trajectories of the previous generation of agents with origin I_L, the long term social belief could be $e_{2\infty}^b$. In this case, expected effort for agent with origin I_L is relatively low. and reference group income will be low if P is low. In this case, relative deprivation leads to lower long term effort level compared to those agents without relative deprivation (agent with origin I_U) and with respect agent with $P > 0$. Observe that in this case there are two possible dynamics. On one hand, when $\mu_0 < \mu_{2\infty}$, the expected effort will increase, and so will individual effort for the future. However, those optimist beliefs have a threshold and steady state of effort belief will be relatively low. On other hand, when $\mu_0 > \mu_{2\infty}$, agents will be attracted with probability $1 - Pr(\mu, \mu_{2\infty})$ by $\mu_{2\infty}$ (and $e_{2\infty}^b$). Because agents believe that their peers (all agents are I_L) in the reference group will decline their effort, their income reference will be lower (relative income effect is lower) and he chooses a lower effort level. This situation determine a "self fulfilling belief" due to effort beliefs.

When the learning function leads to social belief $e_{2\infty}^b$, reference group effect reduces income mobility. Furthermore, because $\overline{\sigma}$ is the true state of the world, the lower effort level for agents with origin I_L would be suboptimal. Although "relative effort pays" and promotes high income mobility, agents with origin I_L are inefficiently discouraged from trying to move up, due to social beliefs, mobility trajectories and inequality.

4.4 An equilibrium analysis without self-motivated agents

In the previous section we assume "self-encouraged agents", therefore the relative deprivation effect always motivates higher effort regardless of effort rewards and ex-

ante inequality. However, Conditions III, IV, VI, and VII, assume that relative effort is a cost, which establishes an ambiguous relationship between effort and reference group income. In this case, ex-ante inequality and circumstances are more relevant to explain the agent response. To be more concrete, we assume that there is a y^{R*}, which holds $\theta\beta_M\Delta y G_{y^{R*}y^{R*}}(y^R*,e) = -G_{y^{R*}e}(y^R*,e)$, and when $E(y^R < y^{R*} \mid I_L) \Rightarrow \theta\beta_M\Delta y G_{y^{R*}y^{R*}}(y^R*,e) < -G_{y^{R*}e}(y^R*,e)$, and when $E(y^R > y^{R*} \mid I_L) \Rightarrow \theta\beta_M\Delta y G_{y^{R*}y^{R*}}(y^R*,e) > -G_{y^{R*}e}(y^R*,e)$. Under this assumption the composition of reference group and ex-ante inequality is even more important for social mobility.

By following analogous reasoning as above, we will arrive to a long term effort level. Under these assumptions, higher expected effort of agent with origin I_L leads to higher steady state effort ($e_{L\infty}$). Therefore, conclusions about previous section remain unchanged. Given P, higher e_∞^b motivates higher effort levels for agents with origin I_L (Observe that $\frac{d(E(y^R|I_L))}{de_L^b} > 0$).

Focus now in the role of ex - ante inequality between social origins, which was measured by $\Delta\pi$. Let $P = 1$ and $\Delta\pi'$ such that $E(y^R < y^{R*} \mid I_L)$. In this case agents with low social origins include individuals from richer origin in their reference group but they face a high relative deprivation. They perceive that the relative costs of effort are too high compared to relative reward. As a result they reduce their effort level in order to avoid frustration. If $\Delta\pi$ is lower, such that $E(y^R > y^{R*} \mid I_L$, relative deprivation might lead to a high effort level. Finally, note that the intensity of this effect is lower when P is low. In this case, a lower P could lead to higher effort level, but that depend on the expected effort of their peers.

Under this assumption there is a non-linear relationship between ex - ante inequality between social origins and effort level of agents with origin. Namely, there is an inverted-U shape relationship between long term effort and $\Delta\pi$.

4.5 Reference group and aspiration failure.

It would be interesting to build a first bridge between reference group theory and the aspiration model proposed by Genicot and Ray (2010). Authors argue that the formation of aspiration is ones of the most relevant factors in explaining upward mobility. They define aspiration as a realistic and attainable target, which, ex-ante, is beyond an agent's possibilities, but which are potentially achievable. They emphasize the role of social interactions and assume that aspirations are based on the cur-

rent and past achievements of an agent's socio-economic neighborhood, which is located within some exogenously given social window ("aspiration window"), defined as $\psi(y_i, D(y_i))$. As a result, an agent's aspirations are determined by his income and the distribution of wealth ($D(y_i)$) in his cognitive window, which could includes his peers or individuals far richer than his. As a result aspiration formation is defined as $a : a(\psi(y_i, D(y_i)))$. Then, they assume that an agent's objective function considers the "aspiration gap" ($ag = y - a$), namely the income difference between his income and his economic aspiration.

$$U(y_i, ag_i) = U(y_i, G(y_i - a(\psi(y_i, D(y_i))) \tag{4.10}$$

Based on these ideas, Ray (2006) identifies two types of aspiration failure. Aspiration failure type I occurs when agents with low social origin do not include agents with high social origin in his aspiration window. As a result, the aspirations gap is low, as will be individual investments for the future. In aspiration failure type II, agents with low social origins include individuals from richer origins in their aspiration window, but the previous inequality and the relative costs of effort are so high, that agents perceive that the goal is unattainable and they are discouraged. As a result they reduce their aspirations and investment level in order to avoid frustration.

If we include a more explicit function of aspiration formation in our model, we can advance in this discussion. If we assume that $a = y^{RG}$, and that P represents the bandwidth of the aspiration window, we could explore the conditions that lead to these types of failures. Furthermore, Ray (2006) argues that an aspiration window depends on how much perceived mobility there is in society, the higher the extent of mobility, the broader the aspirations window. The intergenerational learning proposed in section 4.3 seems adequate to deal with this issue.

On one hand, when individuals are self-motivated, a very low P represents a restricted aspiration window, which leads to aspirations failure type I. In this case, the expected aspiration gap is low, and agents with origin I_L are not encouraged to increase their effort. This will especially be the case if there is economic polarization or other forms of stratification.

On the other hand, there is aspiration failure type II when individuals from I_L include individuals I_U in their "aspiration window" (high P). Failure type II seem less consistent with individuals "self-motivated", although when $P \neq 1$, a low e_∞^b would reduce

effort of agent with origin I_L. When effort beliefs of agents with origin I_L is low, the expected "relative deprivation" will be lower, which induces a decrease in the levels of effort. Although "relative effort pays", agents with origin I_L reduce their effort because they believe that their peers in the reference group will decrease their effort.Therefore, the expected mobility is low (peer's effort "does not pays"), and aspiration gap leads to lower long term effort level compared to those agents with $P = 1$ or a situation with $e^b_{L\infty} = \overline{e}$. This effect will be higher if P is low, which is related with failure type I.

When individuals are not self-motivated reference group may explain naturally failure type II. First, a strong ex-ante inequality between agents with different social origins would lead to lower effort. In this case the relatively individuals poor do aspire to be like the rich, but the income gap is simply too large (see section 4.4). The costs of effort (or investment) is too high, and the reward (in terms of a relative narrowing of the aspiration gap) too low. The reference group leads to aspirations, but the feeling is widespread that such aspirations are largely unreachable. Second, when leisure and relative income are not complements, an agent with social origin I_L is more easily satisfied with his performance and less motivated to achieve high income positions than agents with a less demanding reference group or upper-class origin. As a result, higher reference group income leads to lower effort because agents perceive the goal as unattainable. Therefore, a high relative deprivation reduces the agent's income aspirations and effort level in order to avoid frustration.

5 Conclusion

Our model describes how socio cultural inequalities could partly explain inequality persistence with an emphasis on the role of reference group income.The expected relative deprivation with respect to a reference group affects optimal effort decision. As a result, the reference group effect could play a key role in explaining an additional mechanism of inequality persistence or income mobility. We demonstrate that the magnitudes and direction of these displacements depend on: (a) the direction of the income comparisons (to whom?, "(P)"); (b) its intensity (how much?, α and $G(.)$), (c) ex - ante inequality between agents with different social origins and relative effort reward; and (d) the information about their peers and the past income mobility. Furthermore, this model represents a first bridge between reference group theory and the aspiration failure approach defined by Genicot and Ray (2010). We identify the

conditions under which aspiration failures type I and type II are stables solutions.

(a) The composition of a reference group is relevant regardless of its inheritance pattern. When agents with a low social origin do not include other agents with a high social origin in their reference group, and their peers' expected effort is low, their reference income is closely aligned to their expected income. Therefore they have little incentive to increase their effort, relative deprivation will be low, and so will their investments for the future. This leads to "self fulfilling belief" and determines an aspirations failure type I. However, this effect could be compensated when their peers' expected effort is high.

(b) When agents with origin I_L include other agents with a high social origin in their reference group, their expected income gap, with respect to reference group is higher. In this case, the impact of the relative deprivation effect on effort optimal decision is ambiguous, and assumptions about the functional form of relative concern are keys issues to answer regarding the effect of reference groups on income mobility. Under additive assumption on income comparisons, standard assumption or Prospect theory explain situation in which the effort response (and income mobility) are very different. The former assume self-motivated agents, meanwhile the second assumptions describe discouraged agents. Under non additive assumption, the incidence of reference groups depends on the sign of two functions: G_e(describes how effort affects relative deprivation assessment) and G_{ey^R} (defines if leisure and relative income are complements). If relative income and leisure are complements, the reference group always promotes higher effort levels. In this case, individuals from lower-class backgrounds are self-motivated by a higher income gap between them and their reference group. As a result, lower-class families don't accept an inferior role, and they work harder to pursue personal economic success and "social ascent". In this case, reference group income promotes high income mobility, which is in stark contrast to other models of inequality based upon self-fulfilling beliefs and fatalistic predictions.

(c) However, when relative income and effort are substitutes, the "relative deprivation effect" on effort decisions is ambiguous. In this case, the expected income gap between agent and reference group could encourage or discourage a lower-class agent. Under this scenario, ex - ante inequality and expected relative deprivation are key determinants explaining that ambiguity. There is an inverted – 'U' shape relationship between long term effort and P, and between long term effort and ex-ante inequality. A high reference income could increase effort and ascent mobility, when the income

gap is explained by expected effort of their peers; however, a high ex-ante inequality and low relative effort rewards could reduce the efforts of agent with low social origin. This situation reduces income monbility and it is related with aspiration failure type II.

(d) As the expected reference group income is contextual, their effects depend on how much mobility is perceived. In considering this issue we assume imperfect information and we modeled beliefs using a Bayesian learning process. In this case, there are two stable solutions for effort beliefs, one where agents with origin I_L choose high effort level and an other where their effort levels are low. In the second case, because agents with origin I_L believe that their peers in the reference group will decline their effort, their income reference will be lower and they choose a lower effort level. This situation determine a "self fulfilling belief" due to effort beliefs.

The reference group effect leads to individual decisions on welfare being sub-optimal. When we assume forward-looking agent, this inefficiency is explaining by "between" social origin effect and "within" social origin effect, and it will be higher when $\Delta\pi$ is higher. If we assume backward-looking agents, the results are ambiguous. In this case, even if we assume that "relative effort pays" and promotes high income mobility, agents with origin I_L would be inefficiently discouraged from trying to move up, due to social beliefs, mobility trajectories and inequality.As a result their economic aspirations would be inefficiency lower.

Our conclusions are more general than other models of inequality based upon self-fulfilling beliefs and fatalistic predictions. A more integrated society, one in which there is greater economic diversity in the reference groups and income inequality is relatively low, would open the possibility that the reference group effect increases intergenerational mobility. From a policy perspective, the model suggests, on one hand, that anti-discriminatory and affirmative actions could increase income mobility. Policies which promote an equality of opportunities could mitigate the existence of these type of aspiration failure. On the other hand, higher social cohesion could promote higher intergenerational mobility. For example, if there is residential or public space segregation which reduces the opportunities for social cohesion, public intervention could mitigate this effect. Other possible interventions to close the social origin aspiration gap, could include convening young people and enrolling them in programs away from their communities (Austen – Smith and Fryer, 2005). Ray (2006) suggests some policies for reducing low mobility due to the presence of aspiration failure in

an unequal society. Ray argues that affirmative action or public education could be policy tools to help create local, attainable incentives at the lower end of the wealth or income distribution. On the other hand, if group influence and social interactions are primary determinants of individual aspirations, it may be necessary to ask how redistributive policies can affect group memberships. For example, conditional cash transfer programs to reduce poverty could affect the composition of reference group and change effort decision, which could affect long term income mobility. Finally, the implications in terms of efficiency are ambiguous.

The results of this paper suggest a number of new avenues for empirical research. On one hand, they provide a theoretical framework to evaluate the reaction of agents empirically, in terms of effort, when their relative situation and rewards change. On the other hand, they describe how relative concern could affect income mobility through the formation of aspirations. One problem of general empirical studies on this issue is that they fail to explain the implications of self selection into reference groups. In our model, we avoid discussing this issue and consider the parameters that define reference group integration to be a random variable. Our model demonstrates that reference groups affect income mobility even in this hypothetical situation. However, a model which focuses on endogenizing reference group choice is a possible direction for future research. A number of important issues remain to be addressed. First, our approach assumes only two social origins, but this model can be extended to a model in which society has multiple-social origins. Second, this paper propose an updating belief process, but an additional learning process could be considered. Fourth, the paper only included income relative deprivation, which is one dimension of interpersonal comparison. Finally, in our model the possibility of strategic behavior on the part of agents with different social origins or reference groups is ignored.

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